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Interaction of Global-Scale Atmospheric Vortices: Modeling Based on Hamiltonian Dynamic System of Antipodal Point Vortices on Rotating Sphere

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Abstract

It is shown for the first time that only an antipodal vortex pair (APV) is the elementary singular vortex object on the rotating sphere compatible with the hydrodynamic equations. The exact weak solution of the absolute vorticity equation on the rotating sphere is obtained in the form of Hamiltonian dynamic system for N interacting APVs. This is the first model describing interaction of Barrett vortices corresponding to atmospheric centers of action (ACA). In particular, new steady-state conditions for $N = 2$ are obtained. These analytical conditions are used for the analysis of coupled cyclone-anticyclone ACAs over oceans in the Northern Hemisphere.

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1. Introduction

Investigation of the fluid dynamics on the rotating sphere has fundamental and applied significance for understanding of dominant physical processes in the atmosphere and ocean. Development of such investigations on the base of wave and vortex representations and models is conducted mostly independently. All of them as a rule are based on solving of one and the same equation of absolute vorticity conservation in a thin layer of ideal incompressible fluid on the surface of a rotating sphere.

For corresponding investigations, two directions can be selected. The first one is related with the consideration of the point vortices on the sphere basics of which were founded by Gromeka [1], Zermelo [2] and others. The second direction was initiated by Rossby and is related with the use of Rossby waves [3]. In this paper we present

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results which are linked, though based on the dynamics modeling of point vortices on the sphere, with results of Barrett [4] developing studies initiated by Rossby. For better structuring of the obtained results we give in Section 2 some conclusions of studies on the base of Rossby waves and Barrett vortices. Their modified representation adequate to obtained in Section 3 and further new results is also given. The latter can be used for modeling of quasi-stationary modes and their stability, in particular, for atmospheric centers of action (ACA) using a new approach for description of point vortices on the rotating sphere. The present paper is a more complete version of previous authors' publications [5, 6].

2. Rossby waves and Barrett vortices

Rossby [3] selected long-lived quasi-stationary planetary-scale structures (atmospheric centers of action - ACA) after filtering out pressure field fluctuations related with nonstationary cyclones. To simulate ACA-type modes linear solution of absolute vorticity (ω) equation was used in [3] in the β -plane approximation. Corresponding consideration was conducted in [4, 7 – 9] on the rotating sphere of radius R . Equation of ω conservation in spherical coordinates (r,θ,ϕ) is in this case the following [10] (for $r \approx R$):

$$\frac{D\omega}{Dt} = \frac{\partial\omega}{\partial t} + \frac{V_\theta}{R}\frac{\partial\omega}{\partial\theta} + \frac{V_\varphi}{R\sin\theta}\frac{\partial\omega}{\partial\varphi} = 0\,. \tag{1}$$

Here $\omega = \omega_r + 2\Omega\cos\theta$, Ω – angular velocity of the sphere rotation (for the Earth $\Omega \approx 7.3\cdot 10^{-5}\,\mathrm{sec}^{-1}$), θ – co-latitude, ϕ – longitude; $V_\theta = R\dot{\theta},$ $\dot{\theta} = d\theta / dt,$ $V_\phi = R\sin\theta\dot{\phi},$ $\omega_r = (\partial(V_\phi \sin\theta)/\partial\theta - \partial\phi/\partial\phi)/(R\sin\theta) = -\Delta\psi$ – radial component of the local vortex field on the sphere, Δ – Laplace operator; ψ – stream function for which $V_\phi = -(\partial\psi/\partial\theta)/R,$ $V_\theta = (\partial\psi/\partial\phi)/(R\sin\theta)$ in (1). Equation (1) in the more general case corresponds to the potential vortex conservation, $D(\omega/H)/Dt = 0$, where H – thickness of the fluid layer (see [10]). Equation (1) holds not only for the constant H , but also when H is a Lagrange invariant and velocity field (with zero radial velocity component $\dot{r} = V_r = 0$) is non-divergent, i.e.

$$\mathrm{div}\,\mathbf{V} = \frac{1}{R\sin\theta}(\frac{\partial V_\theta \sin\theta}{\partial\theta} + \frac{\partial V_\varphi}{\partial\varphi}) = -\frac{1}{H}\frac{DH}{Dt} = 0\,.$$

The latter allows to introduce the stream function Ψ . Note that relative (local) vorticity ω_r varies only for the fluid element moving with the change of latitude. Equation (1) is applicable to the flows with any character length scale $L > H$ for $H << R$ [6].

Solution of equation (1), obtained in [4], as well as in [3], in linear approximation (particularly for description of ACA type modes in the Northern Hemisphere – Icelandic and Aleutian Low, and also Azores and Hawaiian High) has the following form

$$\psi = \alpha R^2\cos\theta + \Psi_{\alpha 0}\cos(\beta t + m\varphi)P_n^m(\cos\theta)\,, \tag{2}$$

$$\beta = m(\frac{2(\alpha+\Omega)}{n(n+1)} - \alpha)\,, \tag{3}$$

where n , m – integers, $m \le n, n = 1,2,..;$ P_n^m – adjoint Legendre polynomials. It was assumed in [4] the disturbance amplitude $\Psi_{\alpha 0}$ smallness with respect to the value αR^2 , characterizing zonal flow intensity

$V_{0\phi} = \alpha R \sin\theta$, relative to which wave disturbance of the vortex field is considered on the base of (1).

In [8, 9], it is shown that solution of type (2), (3) preserves its form also for the general case of nonlinear waves when condition $\Psi_{\alpha 0} << \alpha R^2$ already is not necessary.

In [4], solution of the nonlinear equation (1) is obtained generalizing solution (2), (3) (for the case $\alpha = 0$ in (2), (3)) [8, 9]. Meanwhile contrary to (2), (3) the axis of covering all atmosphere global-scale vortex does not coincide as in (2), (3) with the sphere rotation axis. Vortex axis precesses round the sphere axis with rotation from east to west with angular speed $c_1 = -\dfrac{2\Omega}{n(n+1)}$ (Ω – the Earth rotation frequency). Given in [4] solution takes for $\alpha = 0$ and after substitution of $\cos\theta$ by $\cos u_0 = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos(\phi-\phi_0)$ in (2), (3) the following form:

$$\begin{aligned}\psi &= A_1 + B_1[P_n(\cos\theta_0)P_n(\cos\theta) + \\ &+2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}P_n^m(\cos\theta_0)P_n^m(\cos\theta)\cos m(\phi-\phi_0+\frac{2\Omega t}{n(n+1)}] = \\ &= A_1 + B_1 P_n(\cos u_0).\end{aligned} \tag{4}$$

Here θ_0, ϕ_0 – spherical coordinates of the initial axis position for the introduced by Barrett planetary vortex [4] in correspondence with observations [11]. According to [11] circulation of the middle troposphere possesses remarkable eccentricity with the center of symmetry of the flow at a considerable distance from the geographical pole. In such a case the intensity of the zonal component of the flow (with respect to the geographic pole) is low, while the meridional component is large. Harmonic analysis shows that most of the energy of the meridional motion is concentrated in the first longitudinal harmonic (wavelength 360 degrees of longitude) [4]. According to [4] the circulation with a large meridional component may actually be a rather symmetric zonal vortex with respect to an eccentric pole.

As noted in [4], such eccentric planetary-scale vortices previously were not considered in contrast to vortices (with smaller scale) considered by Rossby [12, 13]. For $n = 1$ Barrett planetary vortex axis is static in the absolute coordinate system in which the sphere rotates with angular speed Ω ($c = -\Omega$).

For the further consideration it is convenient to represent the solution of the equation (1) obtained in [4] in the following generalized form (see also [14]):

$$\psi = Y\left(\cos u_0\left(\theta,\ \theta_0,\ \varphi-\varphi_0+\frac{2\Omega t}{\nu(\nu+1)}\right)\right), \tag{5}$$

where Y – eigen-function of Laplace operator, i.e. $\Delta Y = -\dfrac{\nu(\nu+1)}{R^2}Y$. The case of integer eigenvalues $\nu = n$ corresponds to the solution of (4). For arbitrary (including complex) ν the solution of similar type was analyzed in [14] in relation with modeling of blockings on the base of more local dipole vortex structures (modons) constructed with the use of functions Y . In [14] the case was investigated for modeling of local vortices (modons) with Y in (5) defined via Legendre functions of the first and second kind: $P_\nu^\gamma, Q_\nu^\gamma$, i.e.

$$Y(\theta,\phi) = G(\phi)H(\theta), G = e^{\pm i\gamma\phi}, \gamma = m, m = 0, \pm 1, .., \ H(\theta) = \begin{cases} P_\nu^\gamma(\cos\theta) \\ Q_\nu^\gamma(\cos\theta) \end{cases}.$$

It is worth to note that for a stationary case with $\Omega = 0$ (i.e. for static sphere) there exists similar to (5), but even more general form (suggested by E.A. Novikov in 1983) of the solution of (1) as $\psi = F(\cos u_0)$, where ν – arbitrary function of $\cos u_0(\theta, \theta_0, \phi - \phi_0)$ [15]. In [15], in particular, on example of linear function F, corresponding to rigid-body rotation it was considered a model of the global pollution transfer in the field of statistical ACA type vortex ensemble.

It is possible to get modification of solution (5) in which precession frequency of the Barrett vortex axis may be already independent from eigenvalue ν. Let us represent solution of (1) as linear superposition (in the most general form when $\alpha \neq 0$) of the stream function ψ_0 and (5):

$$\psi = \psi_0 + Y + \alpha R^2 \cos\theta \tag{6}$$

Function ψ_0 in (6) corresponds to zero absolute vorticity $\omega = -\Delta\psi_0 + 2\Omega\cos\theta = 0$ and has a form $\psi_0 = -\Omega R^2 \cos\theta$ characterizing rigid-body east to west fluid rotation relative to the sphere surface. In this case the fluid in the absolute coordinate system is static for $\omega = 0$ and $\psi = \psi_0$.

For (6) with any ν in Y the corresponding global vortices axes rotate with the same angular velocity Ω. Further, in Section 3, the modeling of interaction of ACA-type structures corresponding to Barrett planetary vortices is proposed. New approach for description of point vortices on the rotating sphere allows to do it on the basis of model corresponding to the equation of hydrodynamics on the sphere (1).

3. Exact weak solution for the absolute vortex equation

It is possible to search weak solution of the equation (1) in the form of N pairs superposition of point diametrically conjugated vortices on the rotating sphere, hereinafter referred to as antipodal vortices (APV) [16]:

$$\omega = \frac{\Gamma_0}{R^2}(\delta(\theta) - \delta(\theta - \pi)) + \sum_{i=1}^{N} \frac{\Gamma_i}{R^2 \sin\theta_i}(\delta(\theta - \theta_i)\delta(\phi - \phi_i) - \delta(\theta - \pi + \theta_i)\delta(\phi - \phi_i - \pi)), \tag{7}$$

where δ – delta-function. Stream function ψ has a form corresponding to the presence of singularities in 2(N+1) points of the sphere:

$$\psi = \psi_0 + \frac{\Gamma_0}{\pi} Q_0(\cos\theta) + \sum_{i=1}^{N} \frac{\Gamma_i}{\pi} Q_0(\cos u_i(\theta, \phi)), \tag{8}$$

where $Q_0(x) = \frac{1}{2}\ln\frac{1+x}{1-x}$ – Legendre function of the second kind of zero order and zero power, $\cos u_i = \cos\theta\cos\theta_i + \sin\theta\sin\theta_i\cos(\phi - \phi_i)$, and θ_i, ϕ_i – spherical coordinates of the point vortices which may depend on time.

Obviously, the structure of expression (8) exactly corresponds to the particular case of the stream function (6) for $\alpha = 0$ and $m = n = 0$. It follows from (7) that equation $\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta\omega = 0$ holds identically for any values Γ_0 and Γ_i, $i = \overline{1, N}$ and corresponds to the known requirement of equality to zero for the total vorticity on the sphere.

Expression for ψ in (8) takes into account that solution of the equation

$$\omega_{r0} = -\frac{1}{R^2 \sin\theta} \frac{\partial}{\partial\theta} \sin\theta \frac{\partial \psi_{0V}}{\partial\theta} = \frac{\Gamma_0}{R^2}\Big(\delta(\theta) - \delta(\theta - \pi)\Big) \tag{9}$$

is the function $\psi_{0V} = \frac{\Gamma_0}{2\pi} \ln \frac{1+\cos\theta}{1-\cos\theta}$. The last term in (8) is obtained on the base of solving equation (9) using symmetry considerations with substitution $\cos\theta \to \cos u_i(\theta,\phi)$ when vortex axis turns in the direction (θ_i, ϕ_i) from $\theta = 0$.

Vortex field (7) and stream function (8) may be used for getting an exact weak (in the sense of generalized functionals) solution of equation (1). As a result, the form is defined for functions $\theta_i(t)$, $\varphi_i(t)$ which are solutions of the following 2N-dimensional Hamiltonian system of ordinary differential equations ($\dot{\theta}_i = d\theta_i / dt$ and so on):

$$\begin{aligned} \dot{\theta}_i &= \frac{1}{R^2 \sin\theta_i} \frac{\partial \psi(\theta_i, \phi_i)}{\partial \phi_i} = -\frac{1}{\pi R^2} \sum_{k=1, k\neq i}^{N} \frac{\Gamma_k \sin\theta_k \sin(\phi_i - \phi_k)}{1 - \cos^2 u_{ik}}, \\ \sin\theta_i \dot{\phi}_i &= -\frac{1}{R^2} \frac{\partial \psi(\theta_i, \phi_i)}{\partial \theta_i} = -\Omega \sin\theta_i + \frac{\Gamma_0}{\pi R^2 \sin\theta_i} - \\ &- \frac{1}{\pi R^2} \sum_{k=1, k\neq i}^{N} \frac{\Gamma_k (\cos\theta_i \sin\theta_k \cos(\phi_i - \phi_k) - \sin\theta_i \cos\theta_k)}{1 - \cos^2 u_{ik}} . \end{aligned} \tag{10}$$

Here $\Gamma_k = const$ for any Ω and Γ_0, and $\cos u_{ik} = \cos\theta_i \cos\theta_k + \sin\theta_i \sin\theta_k \cos(\phi_i - \phi_k)$. System (10) for $\Omega = 0$ and $\Gamma_0 = 0$ with accuracy up to numerical factor (π) coincides with the corresponding system in [16], where the system is introduced using kinematic considerations (see [2, 17]), but not on the base of exact weak solution of hydrodynamics equation (1). In the case under consideration dissipation processes and energy pumping in the system are treated as not significant or balancing each other. And equations (10), following from (1) and (7), (8), must provide conservation of integral invariants of kinetic energy $\overline{E}$, angular momentum $\overline{\mathbf{M}}$ and impulse $\overline{\mathbf{P}}$, where line above denotes averaging of respective values over the sphere surface. Noted values related to the mass unit in the rotating coordinate system have the following form: $E = \frac{1}{2}\mathbf{V}^2$, $\mathbf{M} = [\mathbf{r} \times \mathbf{V}]$, $\mathbf{P} = \mathbf{V}$, where $\mathbf{V} = d\mathbf{r} / dt$, $\mathbf{r}$ – radius-vector in Cartesian coordinate system (x, y, z), origin of which is in the sphere center. Rotation of the sphere with frequency Ω is performed round axis z and radial motion is absent ($V_r = 0$).

From definition $\overline{E} = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta E = \frac{1}{2} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \psi \omega_r$ it follows $\overline{\mathbf{P}} = 0$, and also

$$\begin{aligned} \overline{E} &= \frac{1}{8\pi} \sum_{i=1}^{N} \sum_{\substack{k=1, \\ i\neq k}}^{N} \frac{\Gamma_i \Gamma_k}{R^2} \ln \frac{1+\cos u_{ik}}{1-\cos u_{ik}} + \frac{\Gamma_0}{2\pi R^2} \sum_{k=1}^{N} \Gamma_k \ln \frac{1+\cos\theta_k}{1-\cos\theta_k} + \\ &+ \frac{4\pi}{3} \Omega^2 R^2 - \Omega(2\Gamma_0 + \sum_{i=1}^{N} \Gamma_i \cos\theta_i) \end{aligned} \tag{11}$$

$$\bar{M}_z = 2\sum_{i=1}^{N}\Gamma_i\cos\theta_i + 4\Gamma_0 - \frac{8\pi\Omega R^2}{3} \quad (12)$$

$$\bar{M}_x = 2\sum_{i=1}^{N}\Gamma_i\cos\phi_i\sin\theta_i,\ \ \bar{M}_y = 2\sum_{i=1}^{N}\Gamma_i\sin\phi_i\sin\theta_i \quad (13)$$

values θ_i and $\phi_i, i=\overline{1,N}$ in (11) – (13) are functions of time defining from dynamic equations (10) under corresponding initial conditions. For $\Omega = 0$ and $\Gamma_0 = 0$ all four values (11) – (13) are invariants of the system (10). It may be checked by differentiating (11) – (13) over time taking into account (10). The invariants exactly coincide with the invariants of the dynamical system in [16 – 18]†. For $\Omega \neq 0$ or $\Gamma_0 \neq 0$ values (13) are already not invariant, but values $\bar{E}$ и $\bar{M}_z$ are still invariants of (10) (it is true for E only if $\Gamma_0 = const$ and $\Omega = const$). Thus, the sphere rotation or accounting of the polar vortices removes degeneration corresponding to the symmetry related with two invariants (13), existing only for $\Omega = 0$ and $\Gamma_0 = 0$. As a result the system (10) has for $\Omega \neq 0$ or $\Gamma_0 \neq 0$ only two independent integral invariants. Moreover, and for $\Omega = \Gamma_0 = 0$ invariants (13) may be not independent from (11), (12).

4. Stationary vortex modes ($N = 2$) and their stability

Let us consider for the system (10) conditions of existence and stability of stationary modes corresponding to equilibria of the vortex pairs for $N = 2$ in (10). Under condition $\dot{\theta}_1 = \dot{\theta}_2 = 0$ equilibrium is possible either for $\phi_1 = \phi_2 = const$, or for $\phi_1 - \phi_2 = \pi$. It corresponds in general case to two distinct vortex stationary modes on the sphere. For example, in the case $\phi_1 = \phi_2$ from the requirement $d(\phi_1 - \phi_2)/dt = 0$ the following condition is obtained:

$$\frac{\gamma_0(\sin^2\theta_{20} - \sin^2\theta_{10})}{\sin\theta_{10}\sin\theta_{20}} = \frac{\gamma_1\sin\theta_{10} + \sin\theta_{20}}{\sin(\theta_{20} - \theta_{10})}, \quad (14)$$

where θ_{10} и θ_{20} – stationary values of θ_1 and θ_2. Additional condition for the absence of absolute motion in this case, according to equality $d(\phi_1 + \phi_2)/dt = 0$, has the form

$$\frac{2\pi R^2\Omega\sin\theta_{10}\sin\theta_{20}}{\Gamma_2} = \frac{\gamma_0(\sin^2\theta_{10} + \sin^2\theta_{20})}{\sin\theta_{10}\sin\theta_{20}} + \frac{\gamma_1\sin\theta_{10} - \sin\theta_{20}}{\sin(\theta_{20} - \theta_{10})}. \quad (15)$$

It is possible to show that for $\gamma_0 = 0$ stationary vortex mode (14), (15) existing for $\gamma_1 = -\frac{\sin\theta_{20}}{\sin\theta_{10}} < 0$, $\Omega R^2/\Gamma_2 < 0$, is stable with respect to small disturbances.

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† Stream function $\psi_1 = \frac{\Gamma_1}{4\pi}\ln\frac{1}{1-\cos\theta}$ used in [16 – 18] for unitary singular vortex does not satisfy hydrodynamics equations in contrast to ψ_{0V} from (9). For corresponding $N > 1$ vortices in [17, 18] their total intensity shall be equal to zero. As a result, these intensities already can't be defined independent of each other in contrast to intensities for APV. For APV correlation between intensities of a vortex and its antipode is initially is installed in the very invariant structure of APV defining direction of rotation axis for Barrett planetary vortex.

For $\gamma_0 \neq 0$ considered stationary state is stable with respect to small disturbances only when the following inequality holds:

$$D \equiv A\gamma_1^2 + 2B\gamma_1 + C < 0, \tag{16}$$

where

$$\theta_{10} \equiv y,\ \theta_{20} \equiv z$$

$$A = \sin^3 y[\sin(2z - y) + \frac{2\sin^2 y \cos z \sin(z-y)}{\sin^2 z - \sin^2 y}], B = \sin y \sin z(\sin^2 z + \sin^2 y)$$

$$C = \sin^3 z[\sin(2y - z) + \frac{2\sin^2 z \cos y \sin(z-y)}{\sin^2 z - \sin^2 y}].$$

Inequality (16) corresponds to the condition of realization for oscillatory mode of small disturbances described by the following system of equations:

$$\frac{d\theta_1^*}{d\tau} = -\Delta\varphi \sin z / \sin^2(z-y),\ \frac{d\Delta\varphi}{d\tau} = -\frac{\theta_1^* D}{\sin^2(z-y)\sin^2 y \sin^3 z},$$

where $\theta_1^*(\tau)$ – disturbance of stationary state *y*, and $\Delta\varphi(\tau)$ – deviation from 0 for the longitude difference $\varphi_1 - \varphi_2$, $\tau = t\Gamma_2 / \pi R^2$.

In the case of Icelandic Low and Azores High in the Northern Hemisphere over Atlantic Ocean the stability condition (16) is reduced for mean values z and y ($z \sim 55°C$, $y \sim 25°C$) (according to data presented in [19] for the period 1949 – 2002) to the following restrictions for the ratio of intensities of the vortices (for $\gamma_1 = -|\gamma_1| < 0$): $1.35 < |\gamma_1| < 5.24$.

5. Estimates of variability (stability) of vortex modes from observations

In the frame of developed in the present paper approach of vortex dynamics analysis on the uniform rotating sphere there was not taken into account the non-uniformity of the underlying surface, non-adiabatic processes and other factors characteristic for real system of atmospheric vortices. Nevertheless, it is possible to estimate significance of contribution of dynamical component of vortex interactions into evolution of large-scale cyclonic and anti-cyclonic ACAs playing important role in the Earth climate system [19 – 21]. A trial to describe ACA dynamics with the use of point vortices was undertaken, in particular, in [18], but without estimation for significance of contribution of different factors on the base of observational data.

One of the factors not taken into account in the proposed above theoretical analysis and which should influence on relative dynamics and stability of ACA vortex system is the mean temperature difference between oceans and continents. It is possible to analyze mutual positions of ACA in the Northern Hemisphere taking into account variations of surface temperature over land and ocean, in particular for winter from CRU data (http://www.uea.ac.uk/cru/data) for 1949 – 2002.

There was conducted analysis of variability of the distance between ACAs by longitude $\Delta\lambda$ for the pairs of Atlantic (Icelandic Low and Azores High) and Pacific (Aleutian Low and Hawaiian High) ACAs as anomality characteristics of the position and instability of the vortex pairs. As a criterion of anomality (instability) of mutual ACA positioning in the particular winter, we take exceeding by the value $\Delta\lambda$ of the corresponding standard deviation with respect to the mean value over all period under consideration (1949 – 2002). Also we estimated degree of anomality (instability) of the temperature difference between land and ocean ΔT in the Northern Hemisphere with respect to the mean mode according to observations for the same period. We estimated probability of correspondence of characteristics of normality (stability) and anomality (instability) for $\Delta\lambda$ and ΔT. For the winter seasons (according to data for 54 winters) for North Atlantic ACA pair the probability of such correspondence is estimated by 67%, and for Pacific ACA pair by 70%.

On the base of the proposed theoretical modeling approach, it was conducted corresponding stability analysis of ACA vortex pairs for the period 1949 – 2002 with the use of estimates of the theoretical parameter γ_1 (ratio of circulations of cyclonic and anticyclonic vortices in the respective ACA pair) and latitude-longitudinal ACA coordinates from data analysis for pressure fields (see [19]). According to theoretical estimates stability or instability for the vortex pair Icelandic Low – Azores High corresponds with probability 78% to empirical estimates of normality or anomality of its longitudinal position ($\Delta\lambda$), and for the vortex pair Aleutian Low – Hawaiian High with somewhat less probability (54%) according to theoretical model.

Obtained estimates show comparability of dynamical and thermal factors in formation of stability modes or instability of the ACA mutual positioning on the sphere.

6. Modeling of blockings and comparison with the results of approximate theoretical approaches (on β-plane and on the rotating sphere)

In relation to the problem of modeling of atmospheric blockings (see [22, 23]) different approaches were proposed for solution of absolute vorticity equation (1) on the rotating sphere [14, 24, 25] and with an approximation of β-plane [26, 27].

6.1. Comparison with β-plane approximation in [26,27]

In [26, 27], there were obtained estimates for dependence of the motion velocity V of the point-vortex pair on their mutual distance d and on intensity Γ of these vortices united in a single dipole vortex structure of modon type (see [14]). To compare with results [26, 27], let us consider dynamic stationary solution of the system (14, 15), which is corresponding to $\varphi_1(t)=\varphi_2(t)=\varphi(t)$, $\theta_1=\theta_{10}=const$, $\theta_2=\theta_{20}=\pi-\theta_{10}=const$, and $V=R\sin\theta_{10}\dot{\varphi}=const$ has a form

$$V=\frac{\Gamma_0}{\pi R\sqrt{1-d^2/4R^2}}-\Omega R\sqrt{1-\frac{d^2}{4R^2}}+\frac{\Gamma_1}{\pi d\sqrt{1-d^2/4R^2}}, \tag{17}$$

where $d=2R\cos\theta_{10}$ – chord distance between point vortices having the same (by absolute value) intensities ($\Gamma_2=-\Gamma_1$ in (14), (15)).

For $\Omega=\Gamma_0\to 0$ value V in (17) coincides with obtained in [26, 27] dependence on d in the limit $d/2R<<1$. At the same time, for finite values of $d/2R$ (even when $\Omega=\Gamma_0=0$) there is significant qualitative difference of the non-monotonic dependence of the value Γ_1 (for fixed velocity V) from noted in [26] monotonic dependence on d. According to (17) (for $\Gamma_0=\Omega=0$) value Γ_1 reaches its maximum value $\Gamma_1=\Gamma_{1\max}=\pi RV$ when $d=d_{\max}=R\sqrt{2}$. Such stationary dipole vortex object moves with the constant velocity from east to west in the case when $\Gamma_1<0$, i.e. when anti-cyclonic vortex is shifted to the pole relative to the cyclonic one (that is characteristic for blockings, in particular of the splitting type [22]). From (17) it follows that under the fixed intensity Γ_1 there can't exist (for $\Gamma_0=\Omega=0$) a dynamic stationary mode with $V<V_{\min}=\Gamma_1/\pi R$. In the β-plane approximation considered in [26, 27], such a conclusion can't be obtained

though it qualitatively agrees with conclusion [27] about existence of maximal allowed distance between vortices in a pair.

6.2. Comparison with approximation taking into account sphere rotation in [24]

For comparison of obtained conclusions on stability for $N = 2$ with conclusions [24], let us introduce a parameter $G = \frac{\bar{M}_{0z}}{\Gamma_0} = \frac{\Gamma_1}{\Gamma_0}\cos\theta_{10} + \frac{\Gamma_2}{\Gamma_0}\cos\theta_{20} = const$ (as $\bar{M}_{0z} = const$). In particular, for $\theta_{20} = \pi - \theta_{10}$ and $\gamma_1 = \frac{\Gamma_1}{\Gamma_2} = -1$ it follows that $\frac{\Gamma_0}{\Gamma_1} = \frac{2\cos\theta_{10}}{G}$. In the case $\theta_{10} < \pi/2$ from the condition of exponential instability $1 + 4\frac{\Gamma_0}{\Gamma_1}\cos\theta_{10} < 0$ we get that instability takes place only for $G < 0$ and $|G| < 8\cos^2\theta_{10} \approx 0.08$ (for $\theta_{10} = 84.3^\circ$ [24]). In [24] similar estimate for the condition of oscillatory instability is obtained: $|G| < 0.1$. In the analyzed case the motion to the west with $\omega < 0$ in contrast to [24] (where it is possible only for $G < 0$) may take place with $\Gamma_1 > 0$ for $G < 0$ and $|G| < 4\cos^2\theta_{10}$, while with $\Gamma_1 < 0$ - for all $G > 0$ (and for $G < 0$ if $|G| > 4\cos^2\theta_{10}$). Hence, conclusions about stability of relational stationary state with $\omega < 0$ (interesting for blocking modeling, in particular), are found to be substantially dependent on $\Gamma_0 \neq 0$. As a result the conclusion about existence of a stability region for stationary vortex modes modeling blockings is obtained in contrast to [24]. This opens the possibility (denied in [24]) of using model of point vortices on the rotating sphere for study of blockings. It is interesting to compare obtained theoretical conclusions with results of analysis of blockings from observations taking into account polar vortices.

6.3. Comparison with an exact stationary solution of equation (1) in [25]

In [25], it is considered dynamically stationary solution of (1) corresponding to zonal flow with constant angular velocity. And solving of equation (1) is reduced to description of value $Q = \Delta\Psi - p\Psi$ (where p is inversely proportional to the constant angular velocity of zonal flow), represented in [25] as linear superposition of N δ-singular objects. Value Q already does not correspond neither local, nor absolute vorticity on the sphere. Therefore in [25] there is no requirement on strict equality to zero of its integral over the sphere surface in contrast to such a requirement for local and absolute vorticity.

7. Conclusions

Considered up to now independently wave and vortex solutions of absolute vorticity equation (1) actually characterize the same hydrodynamic object which can be described by Legendre functions (of the first and second kind), in particular. This object is characterized by dualism wave-vortex when point vortices correspond to the presence in some locations on the sphere of singularity for these functions which are regular for the rest part of the sphere. Here we consider the dynamics of such singularities on the rotating sphere.

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